Installment Loans

Lecture 6
Section 10.4

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Outline

Installment Loans

Definition (Installment Loan)

With an installment loan, the borrower pays back the loan through regular, equal payments (usually monthly). For each payment,

- The interest rate is applied to the outstanding balance at the end of the payment period.
- The interest is added to the outstanding balance.
- And the payment is subtracted from the outstanding balance, producing the new balance to be carried over to the next payment period.

- In order to pay the loan off, the payments must be greater than the interest earned between payments.
- The greater the difference, the faster the loan is paid off.

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  - The bank has "borrowed" a large sum of money from you (your savings account).

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  - You have "borrowed" a large sum of money from the bank (your loan).
  - You write monthly checks to the bank (your payments), paying down that amount.
- Annuity
  - The bank has "borrowed" a large sum of money from you (your savings account).
  - The bank writes monthly checks to you (your withdrawals), paying down that amount.

Installment Loans

- Therefore, the formula for the payment *M* for an installment loan is exactly the same as the annuity formula for drawing down:

$$M = P \left(\frac{r/k}{1 - \left(1 + \frac{r}{k}\right)^{-kt}} \right),$$

where *P* is the principal (amount borrowed), *r* is the annual interest rate, *k* is the number of payments per year, and *t* is the number of years.

- Payments are usually monthly ($k = 12$), but they do not need to be.

Outline

Example

Example (Three-year loan)

- $4,550 is borrowed at 20% for 3 years, with payments made annually.
- Find the annual payment.
- Trace the balance over the three years as interest is applied and the payments are made.

Example

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$$= \frac{(4550)(.20)}{1 - (1.20)^{-3}}$$

$$= \$2160.$$

Example

Example (Three-year loan)

Year	Starting Balance	Interest	Total	Payment	Ending Balance
1	4,550	910	5,460	2,160	3,300

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Example

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Year	Starting Balance	Interest	Total	Payment	Ending Balance
1	4,550	910	5,460	2,160	3,300
2	3,300	660	3,960	2,160	1,800
3	1,800	360	2,160	2,160	0

Example

Example (Financing a Car)

- You would like to purchase a new car priced at $25,000.
- You can put $5,000 down and finance the rest at 6% for 60 months.
- Find
  - The monthly payment.
  - The total amount paid.
  - The total amount of interest paid.

Example

Example (Home Mortgage)

- You would like to purchase a home for $300,000.
- You can put $60,000 down and finance the rest at 3.7% for 30 years, at 3.0% for 15 years, or at 2.9% for 10 years.
- Find
  - The monthly payment for each choice.
  - The total amount paid for each choice.
  - The total amount of interest paid for each choice.

Outline

Assignment

Assignment

- Chapter 10: 57, 58, 59, 61, 62, 63.